Time Reversal and CPT Theorem

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Abstract

This paper proposes a new definition of time reversal centered on Hermitian conjugation (transpose conjugation), aiming to break through the strong intuitive constraint of "physical process reversal" and to reconstruct time reversal as a "dual mirror mapping" of quantum states and operators. Starting from the entropy criterion in classical physics, it argues against the traditional misunderstanding of "motion reversal" and extends to the framework of quantum mechanics. The mathematical self-consistency of the new definition is established through compatibility proofs with Wigner's theorem. By analyzing the relationship between computational complexity, the law of entropy increase, and quantum reversibility, the significant advantages of the new definition in terms of clarity of physical essence, experimental feasibility, and logical simplicity are clarified.

Furthermore, based on the new definition, the symmetry protection mechanisms of the Kramers theorem and topological insulators are re-examined, explicitly proposing the conceptual unbundling of "spin rotation symmetry" from "time-reversal symmetry". This reveals that the essence of traditional topological protection is the synergistic effect of "Hermiticity (universal condition) + specific spin symmetry (specific condition) + topological invariant". Research shows that the new definition strips away redundant physical assumptions from the traditional framework, achieving the unification and clarification of the concept of time reversal across classical and quantum domains, and provides a new analytical framework and perspective for topological states of matter, open quantum systems, and foundational quantum theory.

Keywords: Time-reversal symmetry; Hermitian conjugation; CPT theorem; Kramers theorem; Topological insulator; Spin rotation symmetry; Entropy

1 Introduction

Time-reversal symmetry, as one of the fundamental pillars of physics, has long been deeply bound to the intuition of "reversible backtracking of physical processes". This view, which understands time reversal as the system flowing backward along its original trajectory (e.g., reversing the rotation direction of a spinning top), faces profound contradictions across multidisciplinary fields from classical thermodynamics to quantum information: the core of classical reversibility is entropy conservation, not motion reversal; the extremely high computational complexity of quantum "inverse processes" conflicts with the requirement for simplicity of physical operations; the traditional definition bundles specific operations like spin flip with time reversal itself, leading to conceptual redundancy and logical confusion.

Based on a core insight: **the essence of time reversal should be a dual mapping at the level of mathematical representation of physical systems, rather than a forced reversal of the physical process itself**. Accordingly, we propose the **new definition for the time reversal operator T: T $\equiv$ the operation of Hermitian conjugation (transpose conjugation)**. In quantum systems, its action on operators is $T\hat{O}T^{-1} = \hat{O}^\dagger$, and on state vectors is $T|\psi\rangle = \langle\psi|$ (i.e., ket-bra duality); in classical real-number systems, it naturally reduces to the transpose operation $T\hat{O}T^{-1} = \hat{O}^T$.

This definition discards additional physical constraints such as "spin flip" and "momentum reversal", relying solely on the intrinsic mathematical structure of Hilbert space or real number space. This paper will systematically demonstrate the self-consistency of this definition in both classical and quantum domains, analyze its four major advantages compared to the traditional definition, and focus on showing how it provides a clearer, more modular theoretical framework for understanding key physical issues like the Kramers theorem and topological insulators by **unbundling "time reversal" from "spin rotation symmetry"**.

2 Time Reversal in Classical Physics: Entropy Criterion and Transpose Mapping

2.1 The Fallacy of the Traditional "Motion Reversal" Viewpoint

In classical mechanics, time reversal is often intuitively equated with the "reversal of motion state", e.g., believing that time reversal of a spinning top is simply making it spin in the opposite direction. The fundamental error of this view lies in confusing **state symmetry** with **process reversibility**.

- **The True Criterion for Reversibility is Entropy**: For an ideal frictionless top, its motion process is reversible, but the core reason for this reversibility is that it satisfies Liouville's theorem, conserving phase space volume elements, thereby leading to constant system entropy ($\Delta S = 0$). By applying a dissipationless counter-torque, the system can precisely return to its initial state. This reversibility is **independent of whether the rotation direction is reversed**.
- **"Motion Reversal" Does Not Guarantee Reversibility**: For a real top with friction, even if an external force is used to forcibly reverse its rotation direction, the energy dissipated as heat due to friction cannot spontaneously return, the total system entropy increases ($\Delta S > 0$), and the process is fundamentally irreversible. This proves that there is no necessary connection between "motion state reversal" and the "reversibility" associated with time reversal.

2.2 Classical Time Reversal Under the New Definition: Transpose as Dual Mapping

According to the new definition, classical time reversal is the **transpose transformation** $T : \hat{O} \to \hat{O}^T$. Its physical meaning can be analogized to spatial mirror reflection:

- A mirror does not change the object itself, only generating its left-right dual image. Similarly, classical time reversal (transpose) does not change the physical essence of the system (such as mass, moment of inertia), only performing a dual exchange on the mathematical representation describing the system (e.g., row-column indices of a second-rank tensor).
- The core value of this operation lies in its **mathematical duality**, not in driving the physical process to flow backward. Whether a process is reversible depends on whether it satisfies the condition of **constant total entropy**; the transpose operation is merely a mathematical symmetry exhibited in reversible processes.

2.3 Degeneration of the Classical CPT Theorem: Self-Consistency of the Identity Transformation

Applying the new definition in the classical domain, the combined CPT transformation shows a high degree of simplification and degeneration:

- **C (Charge Conjugation)**: Classical systems lack the concept of particle-antiparticle, degenerating to the identity transformation.

- **T (Time Reversal)**: Defined as transpose $T\hat{O}T^{-1} = \hat{O}^T$.
- **P (Parity)**: Spatial coordinate inversion, in classical tensor descriptions, is often equivalent to an index transpose operation, i.e., $P\hat{O}P^{-1} \sim \hat{O}^T$.

Therefore, the classical combined CPT transformation is: $CTP\hat{O}(CTP)^{-1} \to (\hat{O}^T)^T = \hat{O}$, i.e., it **degenerates to the identity transformation**. This profoundly indicates that, after stripping away the complex particle nature from quantum field theory, classical CPT symmetry merely reflects the **mathematical self-consistency** of classical physical quantities described in the real number domain, and is unrelated to the physical intuition of "time flowing backward" or "space inversion".

3 Traditional Definition of Quantum Time Reversal and Its Limitations

3.1 Traditional Definition: Antiunitary Transformation and Wigner's Theorem

Based on Wigner's theorem, the time reversal operator **T** in quantum mechanics is defined as an **antiunitary transformation**, satisfying:

1. **Antilinearity**: $T(c_1|\psi_1\rangle + c_2|\psi_2\rangle) = c_1^* T|\psi_1\rangle + c_2^* T|\psi_2\rangle$.
2. **Inner Product Preservation**: $\langle T\psi|T\phi\rangle = \langle\psi|\phi\rangle^*$.
3. **For Spin-1/2 Particles**: Its action is additionally stipulated to include spin flip, e.g., $T|\uparrow\rangle = |\downarrow\rangle, T|\downarrow\rangle = -|\uparrow\rangle$, leading to $T^2 = -1$.

This definition aims to make expectation values like $\langle\psi|\hat{p}|\psi\rangle$ under T mimic the classical image of "momentum reversal".

3.2 The Standard CPT Theorem and Its Limitations

The CPT theorem is a cornerstone of quantum field theory, asserting that in any Lorentz-invariant local quantum field theory, the combined transformation of charge conjugation (C), parity (P), and time reversal (T) is an exact symmetry. Its importance lies in guaranteeing the unitarity and causality of the theory. However, the traditional framework has obvious limitations:

1. **Conceptual Bundling and Redundancy**: Strongly coupling time reversal T with charge conjugation C, parity P, and specific operations like spin flip makes the concept of time reversal itself complex and impure.
2. **Extreme Difficulty in Experimental Realization**: Achieving the precise combined operation of "spin flip + momentum reversal + complex conjugation of phases" required by the traditional definition is extremely difficult in experiments.
3. **Conflict with Computational Complexity**: Associating time reversal with the "inverse process" of a physical process, which mathematically corresponds to matrix inversion with a computational complexity as high as $O(N^3)$, violates the intuition that "fundamental physical symmetry operations should be low-complexity and directly observable".

4 The New Definition of Time Reversal: Hermitian Conjugation and Its Advantages

4.1 Core Formulation

- **Quantum Systems**: Time reversal **T** is defined as the **operation of Hermitian conjugation (transpose conjugation)**.
  - For any operator: $T\hat{O}T^{-1} = \hat{O}^\dagger$.
  - For any state vector: $T|\psi\rangle = \langle\psi|$ (realizing the dual mapping from ket space to bra space).
- **Classical Systems**: Naturally reduces to the transpose operation $T\hat{O}T^{-1} = \hat{O}^T$.
- **The CPT Theorem**: C transformation is complex conjugation, P transformation is transpose, T is Hermitian conjugation (transpose conjugation). Clearly, their product is the identity transformation.

4.2 Mathematical Self-Consistency: Compatibility with Wigner's Theorem

The new definition perfectly satisfies Wigner's theorem's requirements for symmetry transformations:

- **Antiunitarity**: The Hermitian conjugation operation is antilinear (because it includes complex conjugation) and satisfies $\langle T\psi|T\phi\rangle = \langle\psi|\phi\rangle^*$, hence it is an antiunitary transformation.
- **Universal Square Property**: For any operator or state vector, $T^2 = 1$. Because $(\hat{O}^\dagger)^\dagger = \hat{O}$, and $(\langle\psi|)^\dagger = |\psi\rangle$. This avoids the spin-dependent split $T^2 = \pm 1$ of the traditional definition, achieving a unified description for all systems.

4.3 The Four Core Advantages of the New Definition

4.3.1 Clearer Physical Essence: Stripping Redundant Constraints

The new definition reduces time reversal to a pure "mathematical dual mapping". The "spin flip" operation that needed to be additionally attached in the traditional definition is found in the new framework to be essentially the natural consequence of the **Hermiticity of spin operators** ($\hat{S}^\dagger = \hat{S}$) under Hermitian conjugation, not a necessary requirement of time reversal itself. The core meaning of time reversal is clarified as the conversion of "observation perspective" or"mathematical description", decoupled from the physical intuition of"time direction".

4.3.2 Lower Computational Complexity: Aligning with Experimental Feasibility

- Hermitian conjugation (or transpose) is an $O(N^2)$ operation, easily implementable in both classical computation and quantum measurement (e.g., choosing a dual measurement basis).
- The traditionally associated"process inversion" (matrix inversion) is a high-complexity $O(N^3)$ operation and is extremely sensitive to errors, making it unsuitable as an operational definition for a fundamental physical symmetry. The computational simplicity of the new definition brings it closer to real physical operations.

4.3.3 Achieving Logical Unification of Classical and Quantum Concepts

The new definition provides a smooth, natural conceptual transition from classical (transpose) to quantum (Hermitian conjugation). Both share the core mathematical connotation of"dual mapping", eliminating the vast conceptual gap between classical reversible transformations and quantum antiunitary transformations in traditional understanding.

4.3.4 Clearly Distinguishing Symmetry from Reversibility Criterion

The new definition establishes a more rigorous logical chain:

- **Time-reversal Symmetry**: A **mathematical symmetry condition**, i.e., the system Hamiltonian satisfies $\hat{H}^\dagger = \hat{H}$ (Hermiticity). This is a universal condition that any observable quantum system must satisfy.
- **Process Reversibility**: A **physical dynamics criterion**, whose core is **entropy conservation** ($\Delta S = 0$). At the quantum level, this corresponds to unitary evolution; at the classical level, it corresponds to phase space conservation under Liouville's theorem.

The new definition clearly states: **Satisfying time-reversal symmetry (Hermiticity) is a necessary condition for a system to be describable, but whether a process is reversible depends on whether the dynamical evolution conserves entropy.** This resolves the confusion of"time reversal = reversibility" in the traditional framework.

5 Reconstruction of Key Physical Systems Under the New Definition: Unbundling Spin Rotation Symmetry

This chapter reconstructs two traditionally closely linked important concepts—the **Kramers theorem** and the **symmetry protection of topological insulators**—based on the new definition. The core is to explicitly point out that traditional theory has unnecessarily bundled "spin rotation symmetry" with "time-reversal symmetry", and the new framework can clearly unbundle them, providing a more modular understanding.

5.1 Reconstruction of the Kramers Theorem

5.1.1 Traditional Framework (Conceptual Bundling)

The traditional Kramers theorem states: If a system satisfies the traditional time-reversal symmetry ($T\hat{H}T^{-1} = \hat{H}$) and $T^2 = -1$ (for spin-1/2 systems), then all energy levels are at least doubly degenerate. Here, the T operation **embeds a 180° spin rotation operation about a specific axis**. Therefore, the "double degeneracy" of this theorem is essentially the result of the **synergistic constraint of "antiunitary time reversal" and "specific spin rotation operation"**. Its physical root is the rotation symmetry in spin space, but it is wrapped within the definition of time reversal.

5.1.2 Under the New Framework (Conceptual Unbundling)

Under the new definition, the action of time reversal **T** (Hermitian conjugation) on spin operators is trivial: $T\hat{S}_iT^{-1} = \hat{S}_i^\dagger = \hat{S}_i$. **T itself no longer contains any active spin rotation.**

Then, the physical fact described by the traditional Kramers theorem—spin-related degeneracy—should be correctly attributed to the system possibly possessing an **independent "spin rotation symmetry"**. For example, if the Hamiltonian is invariant under some spin rotation operation R (e.g., rotation about the z-axis): $R\hat{H}R^{-1} = \hat{H}$, it will lead to degeneracy of states with different spin projections.

5.1.3 Reformulated Expression Under the New Framework

For a quantum system with spin, if its Hamiltonian $\hat{H}$ is Hermitian (i.e., satisfies the new definition's time-reversal symmetry $\hat{H}^\dagger = \hat{H}$), **and** it additionally possesses some kind of **spin rotation symmetry**, then this symmetry will constrain the energy spectrum, possibly leading to degeneracy (e.g., double degeneracy). The new definition's time-reversal symmetry (Hermiticity) is the **universal background** for the system's mathematical self-consistency, while the specific degeneracy pattern is determined by the **independent physical symmetry (spin rotation)**. This achieves clear separation of concepts.

5.2 Restatement of the Symmetry Protection Mechanism in Topological Insulators

5.2.1 Traditional Understanding (Relying on Bundled Concepts)

Time-reversal symmetry-protected topological insulators (e.g., 2D/3D topological insulators) have their stable gapless surface states believed to be protected by traditional time-reversal symmetry ($T^2 = -1$) and the resulting Kramers degeneracy. A magnetic field destroys this symmetry, thereby allowing the surface states to open a gap.

5.2.2 New Framework Understanding (Modular Analysis)

Based on conceptual unbundling, we can perform a more refined modular analysis of the protection mechanism in topological insulators:

1. **Root of Topology**: The existence of topological non-triviality and surface states fundamentally stems from the system's **topological invariant** (e.g., Z_2 invariant), which is a global property relatively independent of the symmetry representation.
2. **Role of Symmetry Protection**: In realizing a specific topological phase like "time-reversal symmetry protected", the system needs to satisfy a set of conditions:
    - **Condition A (Universal Mathematical Self-Consistency)**: The Hamiltonian must be Hermitian, i.e., satisfy the new definition's time-reversal symmetry $\hat{H}^\dagger = \hat{H}$. This is the foundation of any physical system.
    - **Condition B (Specific Physical Symmetry)**: The system must possess some kind of **specific spin rotation symmetry** (e.g., the kind that ensures the surface states realize spin-momentum locking). **This is precisely the part of physics bundled into the T operation in the traditional definition.**
3. **New Interpretation of Magnetic Field Effects**:
    - When an external magnetic field is added (Zeeman term $\hat{H}_Z = \mathbf{B}\cdot\hat{\mathbf{S}}$), the system's Hamiltonian **still satisfies Hermiticity (Condition A)**, therefore the new definition's time-reversal symmetry is not broken.
    - However, the magnetic field **typically breaks the system's original, specific spin rotation symmetry (Condition B)**, because it selects a specific spin polarization direction.
    - **Therefore, what the magnetic field destroys is the specific physical symmetry (Condition B) required to protect that particular topological phase, not the universal mathematical self-consistency (Condition A).** This perfectly explains the experimental phenomenon of magnetic field-driven topological phase transitions, while maintaining the fundamental property of the Hamiltonian as an observable.

5.3 Conclusion

The new framework clearly decomposes the topological protection mechanism into: **Topological invariant (root) + Hermiticity/Mathematical self-consistency (universal background) + Specific spin rotation symmetry (specific protection condition)**. This modular understanding offers greater logical clarity and theoretical flexibility compared to the traditional bundled formulation.

6 Summary and Outlook

This paper systematically proposes and argues for a new theoretical framework defining the time reversal operation as **Hermitian conjugation**. The core innovation of this definition lies in liberating time reversal from the intuitive constraint of "physical process reversal" and repositioning it as the **intrinsic dual mapping of mathematical representation** for quantum and classical systems.

1. **Conceptual Reconstruction and Unification**: The new definition (quantum: Hermitian conjugation; classical: transpose) achieves the logical unification of the concept of time reversal across classical and quantum domains and satisfies the mathematical self-consistency requirements of Wigner's theorem.

2. **Significant Advantages**: The new definition has core advantages such as **clearer physical essence** (stripping redundant operations), **lower computational complexity** (ease of implementation), and **greater logical simplicity** (clearly distinguishing symmetry from reversibility criterion).

3. **Resolving an Internal Contradiction of Traditional Theory**: In traditional theory, a logical contradiction exists between "Hermitian operators correspond to observable physical quantities" and "time reversal inverts some Hermitian operators" — the intrinsic properties of observable physical quantities should not change due to time reversal. The new definition resolves this contradiction.

4. **Conceptual Unbundling**: By clearly **unbundling** the traditionally **bundled together** "time-reversal symmetry (mathematical self-consistency)" and "spin rotation symmetry (specific physics)", the new framework achieves clear conceptual separation. This makes the understanding of the Kramers theorem and the symmetry protection mechanism of topological insulators more modular and precise.